

The πNN and $\pi NN(1535)$ couplings in QCD

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Abstract

We study the two point correlation function of two nucleon currents sandwiched between the vacuum and the pion state. The light cone QCD sum rules are derived for the πNN coupling $g_{\pi NN}$ and the $\pi NN(1535)$ coupling $g_{\pi NN^*}$. The contribution from the excited states and the continuum is subtracted cleanly through the double Borel transform with respect to the two external momenta, $p_1^2, p_2^2 = (p - q)^2$. We first improve the original sum rule for $g_{\pi NN}$ and determine the value of the pion wave function $\varphi_\pi(u_0 = \frac{1}{2}) = 1.5 \pm 0.2$, which is a fundamental nonperturbative quantity like the quark condensate. Our calculation shows that the $\pi NN(1535)$ coupling is strongly suppressed.

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1 Introduction

The πNN coupling $g_{\pi NN}$ and the $\pi NN(1535)$ coupling $g_{\pi NN^*}$ play a very important role in one boson exchange potentials (OBEP) for the nuclear force. Although it is widely accepted that QCD is the underlying theory of the strong interaction, the self-interaction of the gluons causes the infrared behavior and the vacuum of QCD highly nontrivial. In the typical hadronic scale QCD is nonperturbative which makes the first principle calculation $g_{\pi NN}$ and $g_{\pi NN^*}$ unrealistic except the lattice QCD approach, which is very computer time consuming. So a quantitative calculation of the πNN and $\pi NN(1535)$ with a tractable and reliable theoretical approach proves valuable.

The method of QCD sum rules (QSR), as proposed originally by Shifman, Vainshtein, and Zakharov [1] and adopted, or extended, by many others [2, 3, 4], are very useful in extracting the low-lying hadron masses and couplings. In the QCD sum rule approach the nonperturbative QCD effects are partly taken into account through various condensates in the nontrivial QCD vacuum. In this work we shall use the light cone QCD sum rules (LCQSR) to calculate the πNN and $\pi NN(1535)$ couplings.

The LCQSR is quite different from the conventional QSR, which is based on the short-distance operator product expansion. The LCQSR is based on the OPE on the light cone,

which is the expansion over the twists of the operators. The main contribution comes from the lowest twist operator. Matrix elements of nonlocal operators sandwiched between a hadronic state and the vacuum defines the hadron wave functions. When the LCQSR is used to calculate the coupling constant, the double Borel transformation is always invoked so that the excited states and the continuum contribution can be treated quite nicely. Moreover, the final sum rule depends only on the value of the hadron wave function at a specific point, which is much better known than the whole wave function [5]. In the present case our sum rules involve with the pion wave function (PWF) $\varphi_\pi(u_0 = \frac{1}{2})$. Note this parameter is universal in all processes at a given scale. In this respect, $\varphi_\pi(u_0 = \frac{1}{2})$ is a fundamental quantity like the quark condensate, which is to be determined with various nonperturbative methods. Like the quark condensate, it can be determined through the analysis of the light cone sum rules. In [6] the value is obtained as $\varphi_\pi(u_0 = \frac{1}{2}) = 1.2 \pm 0.3$ using the pion nucleon coupling constant and the phenomenological $\rho\omega\pi$ coupling constant as inputs.

The LCQSR has been widely used to derive the couplings of pions with heavy mesons in full QCD [5], in the limit of $m_Q \rightarrow \infty$ [9] and $1/m_Q$ correction [10], the couplings of pions with heavy baryons [11], the ρ decay widths of excited heavy mesons [12], and various semileptonic decays of heavy mesons [13] etc.

The aim of the paper is two-fold. First we improve the original sum rule for $g_{\pi NN}(q^2 = 0)$ [6] in the following way: (1) the contribution from the gluon condensate $\langle g_s^2 G^2 \rangle$ and the quark gluon mixed condensate $\langle g_c \bar{q}\sigma \cdot Gq \rangle$ are calculated, which is numerically not negligible as in the case of the nucleon mass sum rule [3] and the sum rule for the pion nucleon coupling constant [14]; (2) the twist four contribution is collected in a more transparent form and is estimated slightly differently from [6]; (3) the uncertainty due to λ_N is reduced in the numerical analysis with the help of the Ioffe's mass sum rule. We arrive at $\varphi_\pi(1/2) = 1.5 \pm 0.2$ using the experimentally precisely known $g_{\pi NN}$ [15]. Secondly we employ the LCQSR to calculate $\pi NN(1535)$ coupling. The continuum and excited states contribution is subtracted rather cleanly within our approach. Our result shows that the $\pi NN(1535)$ coupling is strongly suppressed. In both cases the final sum rules are stable with reasonable variations of the Borel parameter and the continuum threshold.

Our paper is organized as follows: Section 1 is an introduction. We introduce the two point function for the πNN vertex and saturate it with nucleon intermediate states in section 2. The definitions of the PWFs are presented in section 3. In the following section we present the LCQSR for the πNN coupling and discussions of these PWFs. In section 5 we employ LCQSR to calculate the $\pi NN(1535)$ coupling. A short summary is given in the last section.

2 Two Point Correlation Function for the πNN coupling

The pion nucleon coupling constant $g_{\pi NN}$ has been calculated with the following variations of the traditional QCD sum rule method: (1) the three point correlator of two nucleon and one pseudo-scalar meson interpolating fields, which are saturated with resonances in the nucleon N and pion π channels on the phenomenological side [2, 16, 17, 18]; (2) the two point function of two nucleon interpolating fields sandwiched between the vacuum and one π state and saturating only with N resonances [2, 17, 19]; (3) the external field method, which considers the two point correlator of two nucleon interpolating fields in the presence of the pion field [14]; (4) the light cone QCD sum rules [6]; (5) using the

Goldberger-Treiman relation with the nucleon axial coupling constant g_A determined by QCD sum rules in external fields [7]. The recent calculation of $g_A = 1.37 \pm 0.10$ [8] yields $\frac{g_{\pi NN}^2}{4\pi} = 14.7 \pm 2.1$.

In the method (1) the singular pole structure $\frac{\hat{q}\gamma_5}{q^2}$ was picked out and identified for the extraction of $g_{\pi NN}$. Note the operator product expansion (OPE) for the correlator (1) is valid only in the region $p_1^2 \ll 0$, $p_2^2 \ll 0$, $q^2 \ll 0$. At $q^2 = 0$ the OPE does not hold. Moreover it was pointed out that with the first methods the excited states and continuum contribution was not subtracted away [3, 20], which could contaminate the sum rule severely.

We start with the two point function

$$\Pi(p_1, p_2, q) = \int d^4x e^{ipx} \langle 0 | \mathcal{T} \eta_p(x) \bar{\eta}_n(0) | \pi^+(q) \rangle \quad (1)$$

with $p_1 = p$, $p_2 = p - q$ and the Ioffe nucleon interpolating field [3]

$$\eta_p(x) = \epsilon_{abc} [u^a(x) \mathcal{C} \gamma_\mu u^b(x)] \gamma_5 \gamma^\mu d^c(x), \quad (2)$$

$$\bar{\eta}_p(y) = \epsilon_{abc} [\bar{u}^b(y) \gamma_\nu C \bar{u}^{aT}(y)] \bar{d}^c(y) \gamma^\nu \gamma^5, \quad (3)$$

where a, b, c is the color indices and $\mathcal{C} = i\gamma_2\gamma_0$ is the charge conjugation matrix. For the neutron interpolating field, $u \leftrightarrow d$.

$\Pi(p_1, p_2, q)$ has the general form

$$\Pi(p_1, p_2, q) = F(p_1^2, p_2^2, q^2) \hat{q}\gamma_5 + F_1(p_1^2, p_2^2, q^2) \gamma_5 + F_2(p_1^2, p_2^2, q^2) \hat{p}\gamma_5 + F_3(p_1^2, p_2^2, q^2) \sigma_{\mu\nu} \gamma_5 p^\mu q^\nu \quad (4)$$

It was well known that the sum rules derived from the chiral odd tensor structure yield better results than those from the chiral even ones in the QSR analysis of the nucleon mass and magnetic moment [3, 21]. Most of the QSR analysis of the pion nucleon coupling constant deals with the tensor structure $\hat{q}\gamma_5$. Based on these considerations we shall focus on the same tensor structure and study the function $F(p_1^2, p_2^2, q^2)$ in detail.

The pion nucleon coupling constant $g_{\pi NN}$ is defined by the πN interaction:

$$\mathcal{L}_{\pi NN} = g_{\pi NN} \bar{N} i \gamma_5 \tau \cdot \pi N + h.c., \quad (5)$$

where h.c. stands for the hermitian conjugate.

At the phenomenological level the eq.(36) can be expressed as:

$$\Pi(p_1, p_2, q) = i \lambda_N^2 m_N g_{\pi NN} (q^2) \frac{\gamma_5 \hat{q}}{(p_1^2 - M_N^2)(p_2^2 - M_N^2)} + \dots \quad (6)$$

where we include only the tensor structure $\gamma_5 \hat{q}$ only. The ellipse denotes the continuum and the single pole excited states to nucleon transition contribution. λ_N is the overlapping amplitude of the interpolating current $\eta_N(x)$ with the nucleon state

$$\langle 0 | \eta(0) | N(p) \rangle = \lambda_N u_N(p) \quad (7)$$

3 The Formalism of LCQSR and Pion Wave Functions

Neglecting the four particle component of the pion wave function, the expression for $F(p_1^2, p_2^2, q^2)$ with the tensor structure at the quark level reads,

$$\int e^{ipx} dx \langle 0 | \mathcal{T} \eta_p(x) \bar{\eta}_n(0) | \pi^+(q) \rangle = -4i \epsilon^{abc} \epsilon^{a'b'c'} \gamma_\mu \gamma_5 S_d^{aa'}(x) \gamma_\nu C [\langle 0 | u^b(x) \bar{d}^{b'}(0) | \pi^+(q) \rangle]^T C \gamma_\mu S_u^{cc'}(x) \gamma_5 \gamma_\nu. \quad (8)$$

where $iS(x)$ is the full light quark propagator with both perturbative term and contribution from vacuum fields.

$$\begin{aligned}
iS(x) &= \langle 0|T[q(x), \bar{q}(0)]|0\rangle \\
&= i\frac{\hat{x}}{2\pi^2 x^4} - \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2}{192} \langle \bar{q}g_s \sigma \cdot Gq \rangle \\
&\quad - \frac{ig_s}{16\pi^2} \int_0^1 du \left\{ \frac{\hat{x}}{x^2} \sigma \cdot G(ux) - 4iu \frac{x_\mu}{x^2} G^{\mu\nu}(ux) \gamma_\nu \right\} + \dots
\end{aligned} \tag{9}$$

where we have introduced $\hat{x} \equiv x_\mu \gamma^\mu$. The relevant feynman diagrams are presented in FIG 1. The squares denote the pion wave function (PWF). The broken solid line, broken curly line and a broken solid line with a curly line attached in the middle stands for the quark condensate, gluon condensate and quark gluon mixed condensate respectively.

By the operator expansion on the light-cone the matrix element of the nonlocal operators between the vacuum and pion state defines the two particle pion wave function. Up to twist four the Dirac components of this wave function can be written as [5]:

$$\begin{aligned}
\langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle &= -if_\pi q_\mu \int_0^1 du e^{iuqx} (\varphi_\pi(u) + x^2 g_1(u) + \mathcal{O}(x^4)) \\
&\quad + f_\pi \left(x_\mu - \frac{x^2 q_\mu}{qx}\right) \int_0^1 du e^{iuqx} g_2(u),
\end{aligned} \tag{10}$$

$$\langle \pi(q) | \bar{d}(x) i\gamma_5 u(0) | 0 \rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{iuqx} \varphi_P(u), \tag{11}$$

$$\langle \pi(q) | \bar{d}(x) \sigma_{\mu\nu} \gamma_5 u(0) | 0 \rangle = i(q_\mu x_\nu - q_\nu x_\mu) \frac{f_\pi m_\pi^2}{6(m_u + m_d)} \int_0^1 du e^{iuqx} \varphi_\sigma(u). \tag{12}$$

$$\begin{aligned}
\langle \pi(q) | \bar{d}(x) \sigma_{\alpha\beta} \gamma_5 g_s G_{\mu\nu}(ux) u(0) | 0 \rangle &= \\
if_{3\pi} [(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})] \int \mathcal{D}\alpha_i \varphi_{3\pi}(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)},
\end{aligned} \tag{13}$$

$$\begin{aligned}
\langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vx) u(0) | 0 \rangle &= \\
f_\pi \left[q_\beta \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q \cdot x} \right) - q_\alpha \left(g_{\beta\mu} - \frac{x_\beta q_\mu}{q \cdot x} \right) \right] \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)} \\
+ f_\pi \frac{q_\mu}{q \cdot x} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)}
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
\langle \pi(q) | \bar{d}(x) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx) u(0) | 0 \rangle &= \\
if_\pi \left[q_\beta \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q \cdot x} \right) - q_\alpha \left(g_{\beta\mu} - \frac{x_\beta q_\mu}{q \cdot x} \right) \right] \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)} \\
+ if_\pi \frac{q_\mu}{q \cdot x} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iqx(\alpha_1 + v\alpha_3)}.
\end{aligned} \tag{15}$$

The operator $\tilde{G}_{\alpha\beta}$ is the dual of $G_{\alpha\beta}$: $\tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\delta\rho} G^{\delta\rho}$; $\mathcal{D}\alpha_i$ is defined as $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. Due to the choice of the gauge $x^\mu A_\mu(x) = 0$, the path-ordered gauge factor $P \exp(i g_s \int_0^1 du x^\mu A_\mu(ux))$ has been omitted. The coefficient in front

of the r.h.s. of eqs. (11), (12) can be written in terms of the light quark condensate $\langle \bar{u}u \rangle$ using the PCAC relation: $\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = -\frac{2}{f_\pi^2} \langle \bar{u}u \rangle$.

The PWF $\varphi_\pi(u)$ is associated with the leading twist two operator, $g_1(u)$ and $g_2(u)$ correspond to twist four operators, and $\varphi_P(u)$ and $\varphi_\sigma(u)$ to twist three ones. The function $\varphi_{3\pi}$ is of twist three, while all the PWFs appearing in eqs.(14), (15) are of twist four. The PWFs $\varphi(x_i, \mu)$ (μ is the renormalization point) describe the distribution in longitudinal momenta inside the pion, the parameters x_i ($\sum_i x_i = 1$) representing the fractions of the longitudinal momentum carried by the quark, the antiquark and gluon.

The wave function normalizations immediately follow from the definitions (10)-(15): $\int_0^1 du \varphi_\pi(u) = \int_0^1 du \varphi_\sigma(u) = 1$, $\int_0^1 du g_1(u) = \delta^2/12$, $\int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) = \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) = 0$, $\int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) = -\int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) = \delta^2/3$, with the parameter δ defined by the matrix element: $\langle \pi(q) | \bar{d} g_s \tilde{G}_{\alpha\mu} \gamma^\alpha u | 0 \rangle = i\delta^2 f_\pi q_\mu$.

4 The LCQSR for the πNN coupling

Expressing (8) with the pion wave functions, we arrive at:

$$\begin{aligned} \Pi(p_1, p_2, q) = & \int d^4x \int_0^1 du e^{i(p-uq)x} \left\{ \frac{4f_\pi}{\pi^4 x^6} \left\{ [\varphi_\pi(u) + x^2 g_1(u)] \gamma_5 \hat{q} - i\gamma_5 \left(\hat{x} - \frac{x^2}{q \cdot x} \hat{q} g_2(u) \right) \right\} \right. \\ & - \frac{f_\pi}{9\pi^2 x^4} \mu_\pi \langle \bar{q}q \rangle \varphi_\sigma(u) \gamma_5 (x^2 \hat{q} - q \cdot x \hat{x}) - \frac{f_\pi}{96\pi^2 x^2} \mu_\pi m_0^2 \langle \bar{q}q \rangle \varphi_\sigma(u) \gamma_5 (x^2 \hat{q} - q \cdot x \hat{x}) \\ & \left. + \frac{f_\pi}{192\pi^2 x^2} \langle g_s^2 G^2 \rangle \varphi_\pi(u) \gamma_5 \hat{q} \right\} \\ & - i \int d^4x \int_0^1 du \int \mathcal{D}\alpha_i e^{ip \cdot x - iq \cdot x(\alpha_1 + u\alpha_3)} \frac{f_{3\pi}}{12\pi^2 x^2} \langle \bar{q}q \rangle \varphi_{3\pi}(\alpha_i) [(1-2u)q^2 \hat{x} \\ & + 2(1-2u)(q \cdot x) \hat{q}] + \dots, \end{aligned} \quad (16)$$

where $\mu_\pi = 1.65\text{GeV}$, $f_\pi = 132\text{MeV}$, $\langle \bar{q}q \rangle = -(225\text{MeV})^3$, $\langle \bar{q}g_s \sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = 0.8\text{GeV}^2$, $a = -(2\pi)^2 \langle \bar{q}q \rangle$. We have collected the terms relevant for the tensor structure $\gamma_5 \hat{q}$ in (16) only.

In the following sections we will frequently use integration by parts to absorb the factors $(q \cdot x)$ and $1/(q \cdot x)$, which leads to the derivative and integration of PWFs. For example,

$$\int_0^1 \frac{q \cdot x}{(x^2)^n} \varphi_\pi(u) e^{-iuq \cdot x} du = -i \int \frac{e^{-iuq \cdot x}}{(x^2)^n} \varphi'_\pi(u) du + \varphi_\pi(u) e^{-iuq \cdot x} \Big|_0^1, \quad (17)$$

$$\int_0^1 \frac{e^{-iuq \cdot x}}{q \cdot x} g_2(u) du = -i \int e^{-iuq \cdot x} G_2(u) du - G_2(u) e^{-iuq \cdot x} \Big|_0^1, \quad (18)$$

where the functions $G_2(u)$ is defined as:

$$G_2(u) = - \int_0^u g_2(u) du. \quad (19)$$

Note the second term in (17) and (18) vanishes due to $\varphi_\pi(u_0) = G_2(u_0) = 0$ at end points $u_0 = 0, 1$.

We first finish Fourier transformation in (16). The formulas are:

$$\int \frac{e^{ipx}}{(x^2)^n} d^D x = i(-1)^{n+1} \frac{2^{D-2n} \pi^{D/2}}{(-p^2)^{D/2-n}} \frac{\Gamma(D/2 - n)}{\Gamma(n)}, \quad (20)$$

$$\int \frac{\hat{x}e^{ipx}}{(x^2)^n} d^D x = (-1)^{n+1} \frac{2^{D-2n+1} \pi^{D/2}}{(-p^2)^{D/2+1-n}} \frac{\Gamma(D/2+1-n)}{\Gamma(n)} \hat{p}. \quad (21)$$

Making double Borel transformation with the variables p_1^2 and p_2^2 the single-pole terms in (6) are eliminated. The formula reads:

$$\mathcal{B}_{p_1^2}^{M_1^2} \mathcal{B}_{p_2^2}^{M_2^2} \frac{\Gamma(n)}{[m^2 - (1-u)p_1^2 - up_2^2]^n} = (M^2)^{2-n} e^{-\frac{m^2}{M^2}} \delta(u - u_0). \quad (22)$$

Subtracting the continuum contribution which is modeled by the dispersion integral in the region $s_1, s_2 \geq s_0$, we arrive at:

$$\begin{aligned} & \sqrt{2} m_N \lambda_N^2 g_{\pi NN} e^{-\frac{M_N^2}{M^2}} = \\ & e^{-\frac{u_0(1-u_0)q^2}{M^2}} \left\{ + \frac{f_\pi}{2\pi^2} \varphi_\pi(u_0) M^6 f_2\left(\frac{s_0}{M^2}\right) - \frac{4f_\pi}{\pi^2} [g_1(u_0) + G_2(u_0)] M^4 f_1\left(\frac{s_0}{M^2}\right) \right. \\ & + \frac{f_\pi}{\pi^2} u_0 g_2(u_0) M^4 f_1\left(\frac{s_0}{M^2}\right) + \frac{f_\pi}{9\pi^2} a \mu_\pi [\varphi_\sigma(u_0) + \frac{u_0}{2} \varphi'_\sigma(u_0)] M^2 f_0\left(\frac{s_0}{M^2}\right) \\ & + \frac{f_\pi}{48\pi^2} \langle g_s^2 G^2 \rangle \varphi_\pi(u_0) M^2 f_0\left(\frac{s_0}{M^2}\right) - \frac{f_\pi}{48\pi^2} a m_0^2 \mu_\pi u_0 \varphi'_\sigma(u_0) \\ & \left. + \frac{f_{3\pi}}{6\pi^2} a M^2 f_0\left(\frac{s_0}{M^2}\right) [I_1^G(u_0) + I_2^G(u_0) - 2I_3^G(u_0)] - \frac{f_{3\pi}}{6\pi^2} a q^2 u_0 I_4^G(u_0) \right\}, \quad (23) \end{aligned}$$

where $f_n(x) = 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}$ is the factor used to subtract the continuum, s_0 is the continuum threshold. $u_0 = \frac{M^2}{M_1^2 + M_2^2}$, $M^2 \equiv \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$, M_1^2, M_2^2 are the Borel parameters, and $\varphi'_\pi(u_0) = \frac{d\varphi_\pi(u)}{du}|_{u=u_0}$ etc.

The functions $I_i^G(u_0)$ are defined as:

$$I_1^G(u_0) = \int_0^{u_0} d\alpha_1 \frac{\varphi_{3\pi}(\alpha_1, 1 - u_0, u_0 - \alpha_1)}{u_0 - \alpha_1}, \quad (24)$$

$$I_2^G(u_0) = \int_0^{1-u_0} d\alpha_2 \frac{\varphi_{3\pi}(u_0, 1 - u_0, \alpha_2, 1 - u_0 - \alpha_2)}{1 - u_0 - \alpha_2}, \quad (25)$$

$$I_3^G(u_0) = \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{\varphi_{3\pi}(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2)}{(1 - \alpha_1 - \alpha_2)^2}, \quad (26)$$

$$I_4^G(u_0) = \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{\varphi_{3\pi}(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2)}{(1 - \alpha_1 - \alpha_2)^2} (1 - 2u_0 + \alpha_1 - \alpha_2) u_0. \quad (27)$$

The twist four PWFs $\varphi_\perp(\alpha_i)$, $\varphi_\parallel(\alpha_i)$, $\tilde{\varphi}_\perp(\alpha_i)$ and $\tilde{\varphi}_\parallel(\alpha_i)$ do not contribute to the chiral odd tensor structures, which was first observed in [6]. Moreover our sum rule (23) is symmetric with the Borel parameters M_1^2 and M_2^2 . So it's natural to adopt $M_1^2 = M_2^2 = 2M^2$, $u_0 = \frac{1}{2}$. We shall work in the physical limit $q^2 = m_\pi^2 \rightarrow 0$ and discard the terms with the factor q^2 in (23).

The various parameters which we adopt are $a = 0.55 \text{ GeV}^3$, $\langle g_s^2 G^2 \rangle = 0.48 \text{ GeV}^4$, $m_0^2 = 0.8 \text{ GeV}^2$ at the scale $\mu = 1 \text{ GeV}$, $s_0 = 2.25 \text{ GeV}^2$, $m_N = 0.938 \text{ GeV}$, $\lambda_N = 0.026 \text{ GeV}^3$ [3].

The working interval for analyzing the QCD sum rule (23) is $0.9 \text{ GeV}^2 \leq M_B^2 \leq 1.8 \text{ GeV}^2$, a standard choice for analyzing the various QCD sum rules associated with the

nucleon. In order to diminish the uncertainty due to λ_N , we shall divide (23) by the famous Ioffe's mass sum rule for the nucleon:

$$32\pi^4\lambda_N^2 e^{-\frac{M_N^2}{M^2}} = M^6 f_2\left(\frac{s_0}{M^2}\right) + \frac{b}{4}M^2 f_0\left(\frac{s_0}{M^2}\right) + \frac{4}{3}a^2 - \frac{a^2 m_0^2}{3M^2}. \quad (28)$$

The resulting sum rule depends on the PWFs, the integrals and derivatives of them at the point $u_0 = \frac{1}{2}$. Since δ^2 is numerically small, the uncertainty due to the integral term $G_2(u_0)$ is insignificant. There are many discussions about the leading twist PWF $\varphi_\pi(u)$ in literature [22, 23, 5, 24, 25, 26, 27]. For example, at $u_0 = \frac{1}{2}$ the values of various model functions are: $\varphi_\pi(u_0) = 1.22$ [5], 1.273 [26], 1.25 [25], 1.71 [24], 1.35 [27] and 1.5 for the asymptotic PWF respectively.

Knowledge of the PWFs involved with the gluon field is still very limited. Only the very lowest a few moments of these PWFs were calculated with significant errors using the method of QSR, which in turn were used to determine the detailed forms of the functions. Such an approach is sometimes misleading since there are many wave functions satisfying the same constraints from moments.

In order to illustrate this point more clearly we use the determination of $\varphi_\pi(u)$ as an example. The model wave function for $\varphi_\pi(u)$ based on the QCD sum rule approach was given in [5] as:

$$\varphi_\pi(u, \mu) = 6u\bar{u} \left(1 + a_2(\mu)\frac{3}{2}[5(u - \bar{u})^2 - 1] + a_4(\mu)\frac{15}{8}[21(u - \bar{u})^4 - 14(u - \bar{u})^2 + 1] \right), \quad (29)$$

where $\bar{u} = 1 - u$. Yet the authors pointed out themselves that the oscillations of (29) around $u_0 = \frac{1}{2}$ is unphysical, which is due to the truncation of the series and keeping only the first a few terms when $\varphi_\pi(u)$ is expanded over Gegenbauer polynomials. In [25] it was stressed that: (1) the expansion over Gegenbauer polynomials converges very slowly as can be seen from the large value of a_2 and a_4 ; (2) any oscillating wave function is not physical, since no detached scale is seen to govern such oscillations. Recently Mikhailov and Radyushkin [26] reanalyzed the QCD sum rules for $\varphi_\pi(u)$ taking into account the non-locality of the condensates. They suggested the following wave function:

$$\varphi_\pi(u) = \frac{8}{\pi} \sqrt{u\bar{u}}. \quad (30)$$

Halperin suggested the following form [25]:

$$\varphi_\pi(u) = N \exp\left(-\frac{m^2}{8\beta^2 u\bar{u}}\right) [\mu^2 + \mu\tilde{\mu} + (\tilde{\mu}^2 - 2)u\bar{u}], \quad (31)$$

where $N = 4.53$, $\mu = \frac{m}{2\beta}$, $\tilde{\mu} = \frac{1}{2\beta} \left(\frac{1}{4}m_\pi + \frac{3}{4}m_\rho \right) = \frac{\tilde{m}}{2\beta}$ with $m = 330\text{MeV}$, $\tilde{m} = 620\text{MeV}$, and $\beta = 320\text{MeV}$. Note all the above three forms of PWFs are rather close to the asymptotic form

$$\varphi_\pi^{\text{asym}}(u) = 6u\bar{u}, \quad (32)$$

which are in strong contrast with the original double humped Chernyak-Zhitnitsky form keeping the lowest two orders of the expansion [22]:

$$\varphi_\pi^{\text{asym}}(u) = 30u\bar{u}(1 - 4u\bar{u}). \quad (33)$$

In other words, after summing the whole series the physical PWF can not deviate too much from the asymptotic form. Based on the above consideration we use the asymptotic forms for the PWFs involved with gluons as in [11].

For the other PWFs we use the results given in [5] since they are relatively better known. $\varphi_P(u_0) = 1.19$, $\varphi_\sigma(u_0) = 1.47$, $g_1(u_0) = 0.022\text{GeV}^2$, $g_2(u_0) = 0$ and $G_2(u_0) = 0.02\text{GeV}^2$ at $u_0 = \frac{1}{2}$ and $\mu = 1\text{GeV}$. Their first derivatives satisfy: $\varphi'_\pi(u_0) = \varphi'_P(u_0) = \varphi'_\sigma(u_0) = g'_1(u_0) = G'_2(u_0) = 0$.

The dependence on the Borel parameter M^2 of $g_{\pi NN}$ are shown in FIG 2 with $s_0 = 2.25\text{GeV}^2$ using different values of $\varphi_\pi(u_0)$. From top to bottom the curves corresponds to $\varphi_\pi(u_0) = 1.6, 1.5, 1.4$ respectively. The final sum rule is very stable in the working region of the Borel parameter M^2 and sensitive to the value of $\varphi_\pi(\frac{1}{2})$. Experimentally the πNN coupling constant has been extracted very precisely: $g_{\pi NN} = 13.5$ [15]. Using this value as the input we obtain:

$$\varphi_\pi(u_0, \mu = 1\text{GeV}) = 1.5 \pm 0.2, \quad (34)$$

which is very close to the asymptotic PWF.

It is interesting to notice that the QSR derived from the chiral odd structure $\hat{q}\gamma_5$ yielded $\varphi_\pi(u_0) = 1.6$ while the QSR from the chiral even structure $\hat{q}\hat{p}\gamma_5$ led to a smaller value [6]. After averaging the two values the authors got $\varphi_\pi(u_0) = 1.2 \pm 0.3$. As pointed out in Section 2, generally the QSR derived from chiral odd structure is more reliable. Therefore, such an averaging may not be very suitable here.

Very recently Belyaev and Johnson investigated the relation between light-front quark model and QCD [27]. They found additional support that the two point PWF $\varphi_\pi(u)$ attains the asymptotic form. Numerically $\varphi_\pi(u)$ at $u = \frac{1}{2}$ agrees with the asymptotic PWF within 10%, which was consistent with our result. It appears that almost all the phenomenological analyses share the common feature, i.e., that the PWF starts to approach the asymptotic form more or less at the low scale $\mu = 1\text{GeV}$ already. This point deserves further investigation.

5 The LCQSR for the $\pi NN(1535)$ coupling

Quantum Chromodynamics (QCD) is asymptotically free and its high energy behavior has been tested to one-loop accuracy. On the other hand, the low-energy behavior has become a very active research field in the past years. Various hadronic resonances act as suitable labs for exploring the nonperturbative QCD. The inner structure of nucleon and mesons and their interactions is of central importance in nuclear and particle physics. Internationally there are a number of experimental collaborations, like TJNAL (former CEBAF), COSY, ELSA (Bonn), MAMI (Mainz) and Spring8 (Japan), which will extensively study the excitation of higher nucleon resonances.

Among various baryon resonances, negative parity resonance $N^*(1535)$ is particularly interesting, which dominates the η meson photo- or electro-production on a nucleon. The branching ratio for the decay $N^* \rightarrow \eta N$ is comparable with that for $N^* \rightarrow \pi N$. Considering the phase space difference and using the experimental decay width of N^* [28], we get $g_{\eta NN^*} = 2$ and $g_{\pi NN^*} = 0.7$. The latter value is in strong contrast with the pion nucleon coupling $g_{\pi NN} = 13.4$. Thus arises the question: why is the coupling $g_{\pi NN^*}$ so small compared with $g_{\pi NN}$?

Whether the coupling $g_{\pi NN^*}$ is strongly suppressed is under heated debate in literature. In a recent extended coupled channel analysis of πN scattering, the Jülich group used $g_{\eta NN^*} = 1.94$ and $g_{\pi NN^*} < 0.12$ [29]. Jido, Oka and Hosaka suggested in a recent letter that πNN^* coupling is strongly suppressed as a consequence of chiral symmetry [30]. Their argument was based on a pion-nucleon correlator of two baryon interpolating fields. The chiral transformation properties of their interpolating fields then imply that

the correlator is purely proportional to the tensor structure γ_5 , with no piece of the form $\hat{p}\gamma_5$, which is the relevant structure for πNN^* coupling. Based on the above observation they claimed that the coupling πNN^* vanishes. The above argument was criticized by Birse [31]. He pointed out that the $\hat{p}\gamma_5$ piece of the correlator is a sum of all possible pion-baryon couplings that can contribute. Hence the absence of a $\hat{p}\gamma_5$ piece is a statement about the particular combination of the pion-baryon coupling and the subtraction terms that correspond to the chosen interpolating fields. It does not imply that the physical πNN^* coupling is suppressed. With an interpolating field with covariant derivative for the N^* Kim and Lee used QCD sum rules to estimate $g_{\pi NN^*} \sim 1.5$ [32]. But in their analysis the continuum and excited states contribution is poorly subtracted and the numerical results depend strongly on the value of the quark gluon mix condensate $\langle \bar{q}g_s\sigma \cdot Gq \rangle$, which renders their conclusion less convincing.

In this section we shall employ the LCQSR to calculate πNN^* coupling. The continuum and excited states contribution is subtracted rather cleanly within our approach through double Borel transformation.

We start with the two point function

$$\Pi(p_1, p_2, q) = \int d^4x e^{ipx} \langle 0 | T \eta_p(x; s) \bar{\eta}_n(0; t) | \pi^+(q) \rangle \quad (35)$$

with $p_1 = p$, $p_2 = p - q$ and the general nucleon interpolating field without derivatives which couples to both positive and negative parity nucleon resonances

$$\eta_p(x; s) = \epsilon_{abc} \{ [u^a(x) \mathcal{C} d^b(x)] \gamma_5 u^c(x) + s [u^a(x) \mathcal{C} \gamma_5 d^b(x)] u^c(x) \} \quad (36)$$

where s is the mixing parameter and $\bar{\eta}_p = \eta_p^\dagger \gamma_0$. Note we use a slightly different nucleon interpolating field (36) in this section. The current $\eta_p(x; s = -1)$ is one half of the one defined in 2. Both of them are called Ioffe's current. The later form appears more in literature. The Ioffe's current couples strongly to the positive parity nucleon [3, 33], while it was found that the current $\eta_p(x; s = 0.8)$ is optimized for negative parity nucleons and couples strongly to $N(1535)$ [34].

It is important to note that the diagonal transitions like $N \rightarrow N$, $N^* \rightarrow N^*$ does not contribute to the tensor structure $\hat{p}\gamma_5$. In other words, the function F_2 involves solely with the process $N^* \rightarrow N$ and corresponding continuum contribution. Based on the above observation we shall focus on the chiral odd tensor structure $\hat{p}\gamma_5$ in this section.

The $\pi NN(1535)$ coupling constant $g_{\pi NN^*}$ is defined as:

$$\mathcal{L}_{\pi NN^*} = g_{\pi NN^*} \bar{N} \tau \cdot \pi N + h.c. \quad (37)$$

At the phenomenological level the eq.(35) can be expressed as:

$$\Pi(p_1, p_2, q) = -(m_N + m_{N^*}) g_{\pi NN^*} \left\{ \frac{\lambda_N(s) \lambda_{N^*}(t)}{(p_1^2 - M_N^2)(p_2^2 - M_{N^*}^2)} - \frac{\lambda_{N^*}(s) \lambda_N(t)}{(p_1^2 - M_{N^*}^2)(p_2^2 - M_N^2)} \right\} i \hat{p} \gamma_5 + \dots \quad (38)$$

where we write the structure $\hat{p}\gamma_5$ explicitly only and the continuum contribution is denoted by the ellipse. $\lambda_N(s)$ is the overlapping amplitude of the interpolating current $\eta_N(x)$ with the nucleon state

$$\langle 0 | \eta(0; s) | N(p) \rangle = \lambda_N(s) u_N(p) \quad (39)$$

We neglect the four particle component of the PWF and express (35) with the PWFs. After Fourier transformation and making double Borel transformation with the variables

p_1^2 and p_2^2 the single-pole terms are eliminated and finally we arrive at:

$$\begin{aligned}
& (m_N + m_{N^*})[\lambda_N(s)\lambda_{N^*}(t) - \lambda_{N^*}(s)\lambda_N(t)]g_{\pi NN^*}e^{-(\frac{M_N^2}{M_1^2} + \frac{M_{N^*}^2}{M_2^2})} = \\
& -\frac{f_\pi}{192\pi^2}(1+s)(1+t)\varphi'_\pi(u_0)M^6f_2(\frac{s_1}{M^2}) + \frac{f_\pi}{16\pi^2}(1+s)(1+t)g'_1(u_0)M^4f_1(\frac{s_1}{M^2}) \\
& -\frac{f_\pi}{32\pi^2}[7(1+st) + 3(s+t)]g_2(u_0)M^4f_1(\frac{s_1}{M^2}) + \frac{f_\pi}{288\pi^2}(1+s+t-3st)a\mu_\pi\varphi'_\sigma(u_0)M^2f_0(\frac{s_1}{M^2}) \\
& +\frac{f_\pi}{4608\pi^2}(1+s)(1+t)\langle g_s^2G^2\rangle\varphi'_\pi(u_0)M^2f_0(\frac{s_1}{M^2}) - \frac{f_\pi}{6912\pi^2}(5+3s+3t-11st)am_0^2\mu_\pi\varphi'_\sigma(u_0) \\
& +\frac{f_\pi}{192\pi^2}(s-t)am_0^2\mu_\pi\varphi_P(u_0) - \frac{f_\pi}{2304\pi^2}[19(1+st) + 7(s+t)]\langle g_s^2G^2\rangle g_2(u_0) \\
& -\frac{f_\pi}{1152\pi^2}(1+s)(1+t)\langle g_s^2G^2\rangle g'_1(u_0) \quad (40)
\end{aligned}$$

The twist three PWF $\varphi_{3\pi}$ appears in the combination $f_{3\pi}\varphi_{3\pi}\langle\bar{q}q\rangle q^2$. In the physical limit $q^2 = m_\pi^2 \rightarrow 0$, its contribution is negligible, which is in contrast with the sum rule for $g_{\pi NN}$ (23), where the twist three PWF $\varphi_{3\pi}$ plays an important role.

It is a common practice to adopt the Ioffe current for the ground-state nucleon [3], i.e., letting $s = -1$. Now the equation (40) has a simple form:

$$\begin{aligned}
& (m_N + m_{N^*})[\lambda_N(s)\lambda_{N^*}(t) - \lambda_{N^*}(s)\lambda_N(t)]g_{\pi NN^*}e^{-(\frac{M_N^2}{M_1^2} + \frac{M_{N^*}^2}{M_2^2})} = \\
& -\frac{f_\pi}{8\pi^2}(1-t)g_2(u_0)M^4f_1(\frac{s_1}{M^2}) + \frac{f_\pi}{72\pi^2}ta\mu_\pi\varphi'_\sigma(u_0)M^2f_0(\frac{s_1}{M^2}) \\
& -\frac{f_\pi}{3456\pi^2}(1+7t)am_0^2\mu_\pi\varphi'_\sigma(u_0) - \frac{f_\pi}{192\pi^2}(1+t)am_0^2\mu_\pi\varphi_P(u_0) \\
& -\frac{f_\pi}{192\pi^2}(1-t)\langle g_s^2G^2\rangle g_2(u_0) \quad , \quad (41)
\end{aligned}$$

where $t = 0.8$.

The various parameters are $s_1 = 3.2\text{GeV}^2$, $m_{N^*} = 1.535\text{GeV}$, $\lambda_N(s = -1) = 0.013\text{GeV}^3$, $\lambda_{N^*}(s = 0.8) = 0.27\text{GeV}^3$ with the formulas in [34]. Moreover $\lambda_N(s = -1) \gg \lambda_N(t = 0.8)$, $\lambda_{N^*}(t = 0.8) \gg \lambda_{N^*}(s = -1)$, so it is reasonable to discard the term $\lambda_N(t = 0.8)\lambda_{N^*}(s = -1)$ in the eq. (41). We use the model PWFs presented in [5] to make the numerical analysis.

Our sum rule (41) is asymmetric with the Borel parameters M_1^2 and M_2^2 due to the significant mass difference of N and $N(1535)$. It is natural to let $M_1^2 = 2m_N^2\beta$, $M_2^2 = 2m_{N^*}^2\beta$, where β is a scale factor ranging from 1.0 to 2.0. In this case we have $M^2 = 1.28\beta\text{GeV}^2$, $u_0 = 0.27$, $g_2(u_0) = -0.03\text{GeV}^2$, $\varphi_P(u_0) = 0.66$ and $\varphi'_\sigma(u_0) = 2.63$ at the scale $\mu = 1\text{GeV}$.

The sum rule for $g_{\pi NN^*}$ is very stable with reasonable variations of s_1 and M^2 as can be seen in FIG 3. Finally we have

$$g_{\pi NN^*} = (-)(0.08 \pm 0.03) . \quad (42)$$

Our result is consistent with the conclusions in Refs. [30, 29].

We have included the uncertainty due to the variation of the continuum threshold and the Borel parameter β in (42). In other words, only the errors arising from numerical analysis of the sum rule (41) are considered. Other sources of uncertainty include: (1) the truncation of OPE on the light cone and keeping only the lowest twist operators.

For example the four particle component of PWF is discarded explicitly; (2) the inherent uncertainty due to the detailed shape of PWFs in different models; (3) throwing away the term $\lambda_N(t = 0.8)\lambda_{N^*}(s = -1)$ in the eq. (41); (4) the continuum model etc. In the present case the major error comes from the uncertainty of PWFs since our final sum rule (41) depends both on the value of PWFs and their derivatives at $u_0 = 0.27$. Luckily around $u_0 = 0.27$ different model PWFs have a shape roughly consistent with each other. So a more conservative estimate is to enlarge the error by a factor of two. Now we have

$$g_{\pi NN^*} = (-)(0.08 \pm 0.06) . \quad (43)$$

We want to point out that one should not be too serious about the specific number. What's important is the fact that $|g_{\pi NN^*}| \ll |g_{\pi NN}|$.

6 Discussion

In summary we have constrained the parameter $\varphi_\pi(\frac{1}{2})$ using the experimentally precisely known $g_{\pi NN}$ using LCQSR. We also have also calculated the πNN^* coupling. The continuum and the excited states contribution is subtracted rather cleanly through the double Borel transformation in both cases. Our result shows explicitly the suppression of $g_{\pi NN^*}$.

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Figure Captions

FIG 1. The relevant feynman diagrams for the derivation of the LCQSR for πNN and $\pi NN(1535)$ coupling. The squares denote the pion wave function (PWF). The broken solid line, broken curly line and a broken solid line with a curly line attached in the middle stands for the quark condensate, gluon condensate and quark gluon mixed condensate respectively.

FIG 2. The sum rule for $g_{\pi NN}$ as a functions of the Borel parameter M^2 for different $\varphi_\pi(\frac{1}{2})$ with the continuum threshold $s_0 = 2.25\text{GeV}^2$. From bottom to top the curves correspond to $\varphi_\pi(\frac{1}{2}) = 1.6, 1.5, 1.4$ respectively.

FIG 3. The sum rule for $g_{\pi NN^*}$ as a function of the scale parameter β with the continuum threshold $s_1 = 3.4, 3.2, 3.0 \text{ GeV}^2$ using the PWFs in [5].





